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Two-level Atom in an Electromagnetic Wave of Circle Polarization

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We study a two-level atom interacting with an electromagnetic wave of circle polarization, and work out the wave functions, the energy values and momentum values of the atom.

KEY WORDS: two-level atom; electromagnetic wave of circle polarization.

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1. INTRODUCTION

In recent years, the subject of atomic motion in an electromagnetic wave has attracted much attention because of its important application (Kalin *et al.*, 2003; Arimondo *et al.*, 1981; Cirac *et al.*, 1994; Cook, 1979; Cook *et al.*, 1985; Cook and Bernhardt, 1978; Dalibard and Tannoudji, 1989; Doery *et al.*, 1995; Marte *et al.*, 1994; Mittleman *et al.*, 1977; Stenhdm, 1986; Wineland and Itano, 1979; Yariv, 1989). We have studied a two-level atom interacting with a very weak standing electromagnetic wave (Kalin *et al.*, 2003). In this paper, we study the motion of a two-level atom in an electromagnetic wave of circle polarization, and work out the wave functions, the energy values and momentum values of the atom. The intensity of the electromagnetic wave of circle polarization can be arbitrary.

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2. SCHRODINGER'S EQUATION OF A TWO-LEVEL ATOM IN AN ELECTROMAGNETIC WAVE OF CIRCLE POLARIZATION

We consider a two-level atom of mass m, dipole moment $\vec{D}$, which starts out moving in the z-direction with momentum p_0, then is irradiated by an electromagnetic wave of circle polarization with a wave vector k and an angular frequency ω_L. The electromagnetic wave of circle polarization propagates along the positive z-direction. According to the theory of electromagnetic wave and electromagnetic field, the electronic field $\vec{E}$ of the electromagnetic wave of circle polarization is assumed to be the form.

$$\vec{E} = E_x\vec{i} + E_y\vec{j} \tag{1}$$

$$E_x = A\cos(\omega_L t - kz) \tag{2}$$

$$E_y = A\sin(\omega_L t - kz) \tag{3}$$

where A is the amplitude of $\vec{E}$.

The Hamiltonian of a two-level atom interacting with an electromagnetic wave of circle polarization is given by

$$\widehat{H} = \frac{P^2}{2m} + \frac{1}{2}\,\hbar\,\omega\sigma_3 + V \tag{4}$$

$$V = -\vec{D}\bullet\vec{E} \tag{5}$$

where $\frac{p^2}{2m}$ is the kinetic energy associated with the center-of-mass momentum along the z-direction, $\frac{1}{2}\,\hbar\,\omega\sigma_3$ is the Hamiltonian associated with the internal motion of the atom, and V is dipole interacting energy between the atom and the electromagnetic wave of circle polarization.

We denote the ground state vector and excited vector of the two-level atom by $|2\rangle$ and $|1\rangle$. According to the feature of the dipole transition (Yariv, 1989), the action of V on these state vectors are

$$V|1\rangle = v_{21}|2\rangle \quad \text{and} \quad V|2\rangle = v_{12}|1\rangle \tag{6}$$

where $v_{12} = \langle 1|\,V\,|2\rangle$ and $v_{21} = \langle 2|\,V\,|1\rangle$ are the nonzero matrix elements of V.

Setting $D^{\pm} = D_x \pm i D_y$ and $E^{\pm} = E_x \pm i E_y$, V may be written as

$$V = -\frac{1}{2}(D^+E^- + D^-E^+) \tag{7}$$

Using

$$D_{12}^- = \langle 1|\,D^-\,|2\rangle = D_{21}^+ = \langle 2|\,D^+\,|1\rangle = 0 \tag{8}$$

we have

$$D_{12}^{+} = D_{21}^{-} = \langle 1|\, D^{+}\, |2\rangle = 2\,\langle 1|\, D_x\, |2\rangle = 2D \tag{9}$$

where

$$D = \langle 1|\, D_x\, |2\rangle \tag{10}$$

then we have

$$v_{12} = -DE^{-} = -D(E_x - iE_y) \tag{11}$$

$$v_{21} = -DE^{+} = -D(E_x + iE_y) \tag{12}$$

using Eqs. (2, 3, 11 and 12), we have

$$v_{12} = -\,\hbar\,\Omega e^{i(\omega_L t - kz)} \quad \text{and} \quad v_{21} = -\,\hbar\,\Omega e^{-i(\omega_L t - kz)} \tag{13}$$

where $\hbar\Omega = DA$,Ω is called induced rat, which describes the interaction intensity.

In order to study the motion of the system, we solve the Schrodinger equation

$$i\hbar \frac{d}{dt}\,|\varphi\rangle = \widehat{H}\,|\varphi\rangle \tag{14}$$

for an arbitrary state $|\varphi\rangle$.

Expanding the state $|\varphi\rangle$ in terms of $|2\rangle$ and $|1\rangle$, we have

$$|\varphi\rangle = \varphi_1(z,t)\,|1\rangle + \varphi_2(z,t)\,|2\rangle \tag{15}$$

Using Eqs. (4 , 7, 13 and 15), Eq. (14) is reduced to

$$i\hbar \frac{d}{dt}\varphi_1 = \left(\frac{p^2}{2m} + \frac{1}{2}\,\hbar\,\omega\right)\varphi_1 - \,\hbar\,\Omega e^{-i(\omega_L t - kz)}\varphi_2 \tag{16}$$

$$i\hbar \frac{d}{dt}\varphi_2 = \left(\frac{p^2}{2m} - \frac{1}{2}\,\hbar\,\omega\right)\varphi_2 - \,\hbar\,\Omega e^{-i(\omega_L t - kz)}\varphi_1 \tag{17}$$

3. SOLUTION OF THE SCHRODINGER EQUATION

Eliminating φ_1 from Eqs. (16) and (17)

$$\varphi_1 = \frac{\mathrm{i}\,\hbar\,\frac{d\varphi_2}{dt} - \left(\frac{p^2}{2m} - \frac{1}{2}\,\hbar\,\omega\right)\varphi_2}{\hbar\,\Omega e^{i(\omega_L t - kz)}}\; i\hbar\,\frac{d}{dt}\,\frac{i\hbar\,\frac{d\varphi_2}{dt} - \left(\frac{p^2}{2m} - \frac{1}{2}\,\hbar\,\omega\right)\varphi_2}{\hbar\,\Omega e^{i(\omega_L t - kz)}}$$

$$= \left(\frac{p^2}{2m} + \frac{1}{2}\,\hbar\,\omega\right) i\hbar\,\frac{i\hbar\,\frac{d\varphi_2}{dt} - \left(\frac{p^2}{2m} - \frac{1}{2}\,\hbar\,\omega\right)\varphi_2}{\hbar\,\Omega e^{i(\omega_L t - kz)}} - \,\hbar\,\omega e^{-i(\omega_L t - kz)}$$

$$-\hbar^2\frac{d^2\varphi_2}{dt^2} - i\hbar\left(\frac{p^2}{2m} - \frac{1}{2}\hbar\omega\right)\frac{d\varphi_2}{dt} + i\hbar^2\omega_L\frac{d\varphi_2}{dt} - \hbar\omega_L\left(\frac{p^2}{2m} - \frac{1}{2}\,\hbar\,\omega\right)\varphi_2$$

$$= i\hbar\left(\frac{p^2}{2m} + \frac{1}{2}\hbar\omega\right)\frac{d\varphi_2}{dt} - \left(\frac{p^2}{2m} + \frac{1}{2}\hbar\omega\right) \times \left(\frac{p^2}{2m} - \frac{1}{2}\,\hbar\,\omega\right)\varphi_2 - \,\hbar^2\Omega^2\varphi_2$$

$$-\,\hbar^2\frac{d^2\varphi_2}{dt^2} - i\hbar\,\left(\frac{p^2}{m} - \,\hbar\,\omega_L\right)\frac{d\varphi_2}{dt} - \left(\frac{p^2}{2m} - \frac{1}{2}\,\hbar\,\omega\right)$$

$$\times\left(\hbar\,\omega_L - \frac{p^2}{2m} - \frac{1}{2}\,\hbar\,\omega\right)\varphi_2 + \,\hbar^2\Omega^2\varphi_2 = 0$$

from which we obtain the equation for φ_2

$$\frac{d^2\varphi_2}{dt^2} + i\frac{1}{\hbar}\left(\frac{p^2}{m} - \,\hbar\,\omega_L\right)\frac{d\varphi_2}{dt}$$
$$+\,\frac{1}{\hbar^2}\left[\left(\frac{p^2}{2m} - \frac{1}{2}\,\hbar\,\omega\right)\left(\hbar\,\omega_L - \frac{1}{2}\,\hbar\,\omega - \frac{p^2}{2m}\right) + \,\hbar^2\Omega^2\right]\varphi_2 = 0 \quad (18)$$

We expand φ_2 as

$$\varphi_2 = \sum_{n=0}^{\infty} C_n(t)e^{i(P_0+n\,\hbar\,k)z/\,\hbar} \quad (19)$$

where p_0 is the initial center-of-mass momentum of the two-level atom. Substituting Eq. (19) into (18), we have

$$C''_n(t) + i\frac{1}{\hbar}\left[\frac{(p_0+n\,\hbar\,k)^2}{m} - \,\hbar\,\omega_L\right]C'_n(t) + \frac{1}{\hbar^2}\left\{\left[\frac{(p_0+n\,\hbar\,k)^2}{2m} - \frac{1}{2}\,\hbar\,\omega\right]\right.$$
$$\left.\times\left[\,\hbar\,\omega_L - \frac{1}{2}\,\hbar\,\omega - \frac{(p_0+n\,\hbar\,k)^2}{2m}\right] + \,\hbar^2\Omega^2\right\}C_n(t) = 0 \quad (20)$$

For convenience, we rewrite Eq. (20) as

$$C''_n(t) + ia_1(n)C'_n(t) + a_2(n)C_n(t) = 0 \quad (21)$$

where

$$a_1(n) = \frac{1}{\hbar}\left[\frac{(p_0+n\,\hbar\,k)^2}{m} - \,\hbar\,\omega_L\right]$$

$$a_2(n) = \frac{1}{\hbar^2}\left\{\left[\frac{(p_0+n\,\hbar\,k)^2}{2m} - \frac{1}{2}\,\hbar\,\omega\right]\right.$$
$$\left.\times\left[\hbar\,\omega_L - \frac{1}{2}\,\hbar\,\omega - \frac{(p_0+n\,\hbar\,k)^2}{2m}\right] + \,\hbar^2\Omega^2\right\} \quad (22)$$

Equation (21) can be written as

$$C_0''(t) + ia_1(0)C_0'(t) + a_2(0)C_0(t) = 0 \tag{23}$$

$$C_1''(t) + ia_1(1)C_1'(t) + a_2(1)C_1(t) = 0 \tag{24}$$

$$C_2''(t) + ia_1(2)C_2'(t) + a_2(2)C_2(t) = 0 \tag{25}$$

$$C_3''(t) + ia_1(2)C_3'(t) + a_2(2)C_3(t) = 0 \tag{26}$$

Solving Eq. (23), we can obtain

$$C_0(t) = A_0 e^{\lambda_1 t} + B_0 e^{\lambda_2 t} \tag{27}$$

where

$$\lambda_1 = -\frac{i}{\hbar}E_2^1, \quad \lambda_2 = -\frac{i}{\hbar}E_2^2 \tag{28}$$

$$\lambda^2 + ia_1(0)\lambda + a_2(0) = 0$$

$$\lambda_{1,2} = \frac{-ia_1(0) \pm \sqrt{[ia_1(0)]^2 - 4a_2(0)}}{2}$$

$$[ia_1(0)]^2 = -\frac{1}{\hbar^2}\left(\frac{p^2}{m} - \hbar\,\omega_L\right)^2 = -\frac{1}{\hbar^2}\left(\frac{p^4}{m^2} + \hbar^2\omega_L^2 - \frac{2p^2}{m}\hbar\,\omega_L\right)$$

$$\Delta = -\frac{1}{\hbar^2}\left(\frac{p_0^4}{m^2} + \hbar^2\omega_L^2 - \frac{2p_0^2}{m}\hbar\omega_L + \frac{2p_0^2}{m}\hbar\omega_L - \frac{p_0^4}{m^2} - 2\hbar^2\omega\omega_L + \hbar^2\omega^2 + 4\,\hbar^2\Omega^2\right)$$
$$= -\frac{\hbar^2}{\hbar^2}\left[4\Omega^2 + (\omega - \omega_L)^2\right]$$

$$\lambda_{1,2} = -\frac{i}{\hbar}\left[\left(\frac{p_0^2}{2m} - \frac{1}{2}\hbar\omega\right) \pm \frac{\hbar}{2}\sqrt{4\Omega^2 + (\omega - \omega_L)^2}\right]$$

Now we can obtain

$$E_2^1 = \frac{p_0^2}{2m} - \frac{1}{2}\hbar\omega_L + \frac{\hbar}{2}\sqrt{4\Omega^2 + (\omega - \omega_L)^2} \tag{29}$$

$$E_2^2 = \frac{p_0^2}{2m} - \frac{1}{2}\hbar\omega_L - \frac{\hbar}{2}\sqrt{4\Omega^2 + (\omega - \omega_L)^2} \tag{30}$$

And the solution of Eq. (24) is

$$C_1(t) = A_1 e^{-iE_2^3 t/\hbar} + B_1 e^{-iE_2^4 t/\hbar} \tag{31}$$

$$E_2^3 = \frac{(p_0 + \hbar k)^2}{2m} - \frac{1}{2}\hbar\omega_L + \frac{\hbar}{2}\sqrt{4\Omega^2 + (\omega - \omega_L)^2} \tag{32}$$

$$E_2^4 = \frac{(p_0 + \hbar k)^2}{2m} - \frac{1}{2}\hbar\omega_L - \frac{\hbar}{2}\sqrt{4\Omega^2 + (\omega - \omega_L)^2} \tag{33}$$

And the solution of Eq. (25) is

$$C_2(t) = A_2 e^{-iE_2^5 t/\hbar} + B_2 e^{-iE_2^6 t/\hbar} \tag{34}$$

$$E_2^5 = \frac{(p_0 + 2\hbar k)^2}{2m} - \frac{1}{2}\hbar\omega_L + \frac{\hbar}{2}\sqrt{4\Omega^2 + (\omega - \omega_L)^2} \tag{35}$$

$$E_2^6 = \frac{(p_0 + 2\hbar k)^2}{2m} - \frac{1}{2}\hbar\omega_L - \frac{\hbar}{2}\sqrt{4\Omega^2 + (\omega - \omega_L)^2} \tag{36}$$

where A_0, B_0, A_1, B_1, A_2 and B_2 are determined by the initial conditions.

In terms of Eqs. (29, 30, 32, 33, 35 and 36), it is obtained easily that ground state energy level

$$E_2 = \frac{p_0^2}{2m} - \frac{1}{2}\hbar\omega \tag{37}$$

of the two-level atom is split into E_2^1, E_2^2, E_2^3, E_2^4, E_2^5, E_2^6, ... E_2^1 and E_2^2 are the ground state energy levels of the two-level atom that its center-of-mass momentum is p_0. E_2^3 and E_2^4 are the ground state energy levels of the two-level atom that its center-of-mass momentum is $(p_0 + \hbar k)$. E_2^5 and E_2^6 are the ground state energy levels of the two-level atom that its center-of-mass momentum is $(p_0 + 2\hbar k)$, and so on. When the two-level atom interacts with an electromagnetic wave of circle polarization, its center-of-mass momentum is quantization, the only possible energy levels of its center-of-mass momentum are $(p_0 + j\hbar k)$, where $j = 1, 2, 3, \ldots$. This result can explain Bragg scattering phenomenon. It will be applied in quantum computation.

Eliminating φ_2 from Eqs. (16) and (17), we obtain the equation for φ_1

$$\frac{d^2\varphi_1}{dt^2} + i\frac{1}{\hbar}\left(\frac{p^2}{m} + \hbar\omega_L\right)\frac{d\varphi_1}{dt} + \frac{1}{\hbar^2}\left[\left(\frac{p^2}{2m} + \frac{1}{2}\hbar\omega\right)\left(-\hbar\omega_L + \frac{1}{2}\hbar\omega - \frac{p^2}{2m}\right) + \hbar^2\Omega^2\right]\varphi_1 = 0 \tag{38}$$

We expand φ_1 as

$$\varphi_1 = \sum_{n=0}^{\infty} b_n(t) e^{i(P_0+n\hbar k)z/\hbar} \tag{39}$$

Substituting Eq. (51) into (50), we have

$$b_n''(t) + i\beta_1(n) b_n'(t) + \beta_2(n) b_n(t) = 0 \tag{40}$$

where

$$\beta_1(n) = \frac{1}{\hbar}\left[\frac{(p_0+n\hbar k)^2}{m} + \hbar\omega_L\right]$$

$$\beta_2(n) = \frac{1}{\hbar^2}\left\{\left[\frac{(p_0+n\hbar k)^2}{2m} + \frac{1}{2}\hbar\omega_L\right] \times \left[-\hbar\omega_L + \frac{1}{2}\hbar\omega - \frac{(p_0+n\hbar k)^2}{2m}\right] + \hbar^2\Omega^2\right\} \tag{41}$$

Solving Eq. (21) by using the same approach as before, we can obtain the solution of Eq. (40)

$$b_0(t) = A_0' e^{-iE_1^1 t/\hbar} + B_0' e^{-iE_1^2 t/\hbar} \tag{42}$$

$$E_1^1 = \frac{p_0^2}{2m} + \frac{1}{2}\hbar\omega_L + \frac{\hbar}{2}\sqrt{4\Omega^2 + (\omega-\omega_L)^2} \tag{43}$$

$$E_1^2 = \frac{p_0^2}{2m} + \frac{1}{2}\hbar\omega_L - \frac{\hbar}{2}\sqrt{4\Omega^2 + (\omega-\omega_L)^2} \tag{44}$$

And

$$b_1(t) = A_1' e^{-iE_1^3 t/\hbar} + B_1' e^{-iE_1^4 t/\hbar} \tag{45}$$

$$E_1^3 = \frac{(p_0+\hbar k)^2}{2m} + \frac{1}{2}\hbar\omega_L + \frac{\hbar}{2}\sqrt{4\Omega^2 + (\omega-\omega_L)^2} \tag{46}$$

$$E_1^4 = \frac{(p_0+\hbar k)^2}{2m} + \frac{1}{2}\hbar\omega_L - \frac{\hbar}{2}\sqrt{4\Omega^2 + (\omega-\omega_L)^2} \tag{47}$$

And

$$b_2(t) = A_2' e^{-iE_1^5 t/\hbar} + B_2' e^{-iE_1^6 t/\hbar} \tag{48}$$

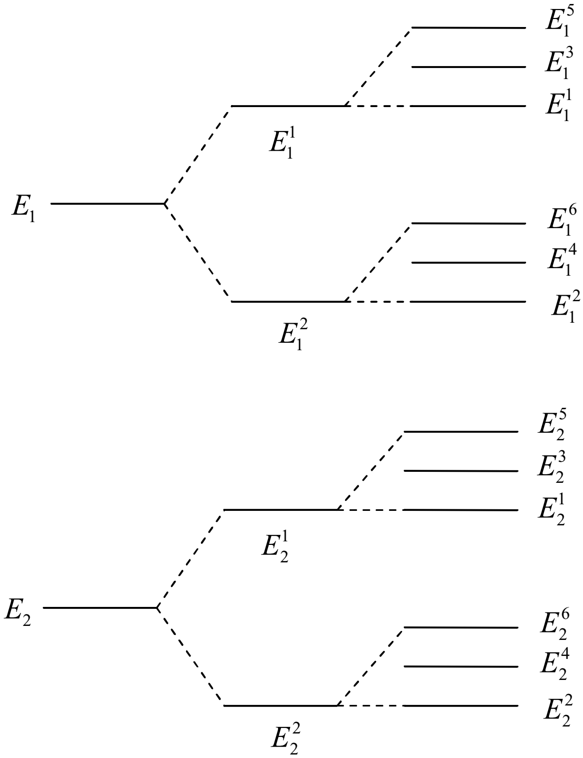
$$E_1^5 = \frac{(p_0+2\hbar k)^2}{2m} + \frac{1}{2}\hbar\omega_L + \frac{\hbar}{2}\sqrt{2\Omega^2 + (\omega+\omega_L)^2} \tag{49}$$

$$E_1^6 = \frac{(p_0+2\hbar k)^2}{2m} + \frac{1}{2}\hbar\omega_L - \frac{\hbar}{2}\sqrt{4\Omega^2 + (\omega+\omega_L)^2} \tag{50}$$

The constants A'_0, B'_0, A'_1, B'_1, A'_2, B'_2, ... are determined by the initial conditions.

4. DISCUSSION

We can draw the energy levels figure by the value of E_2^1, E_2^2, E_2^3, E_2^4, E_2^5, E_2^6, ... and E_1^1, E_1^2, E_1^3, E_1^4, E_1^5, E_1^6 ...



The above figure denotes that the ground state energy level E_2 and the excited state energy level E_1 of the two-level atom are split into E_2^1,E_2^2 and E_1^1,E_1^2 due to the coupling between the dipole of the atom and the electromagnetic wave of circle polarization. The center-of-mass momentum is changed quantitatively due to the collision between the photon and the atom,the energy levels are split further into E_2^1, E_2^2, E_2^3, E_2^4, E_2^5, E_2^6, ... and E_1^1, E_1^2, E_1^3, E_1^4, E_1^5, E_1^6 .... The intensity of the electromagnetic wave of circle polarization is arbitrary.

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REFERENCES

Arimondo, E., Bambini, A., and Stenholm, S. (1981). *Physical Review A* **24**, 898.
Cirac, J. I., Garay, L. J., Blatt, R., Parkins, A. S., and Zoller, P. (1994). *Physical Review A* **49**, 421.
Cook, R. J. (1979). *Physical Review A* **20**, 224.
Cook, R. J. and Bemhardt, A. F. (1978). *Physical Review A* **18**, 2533.
Cook, R. J., Shankland, D. G., and Walls, A. L. (1985). *Physical Review A* **31**, 564.
Dalibard, J. and Tannoudji, C. C. (1989). *Journal of the Optical Society of America B* **6**, 2023.
Doery, M. R., Vredenbregt, E. J., and Bergeman, T. (1995). *Physical Review A* **51**, 4881.
Marte, P., Dum, R., Tareb, R., Zoller, P., Shahn, M. S., and Pretiss, M. (1004). *Physical Review A* **49**, 4826.
Mittleman, M. H., Rubin, K., Callender, R. H., and Gersten, J. I. (1977). *Physical Review A* **16**, 583.
Stenhdm, S. (1986). *Review of Modern Physics* **58**, 699.
Su, K., Luo, W., Xu, X., and Yang, K. (2003). *International Journal of Theoretical Physics* **43**, 1641.
Wineland, D. J., and Itano, W. M. (1979). *Physical Review A* **20**, 1521.